



Munich Personal RePEc Archive

# **The perils of first-order conditions of New Keynesian models**

Minseong Kim

14 April 2016

Online at <https://mpra.ub.uni-muenchen.de/71225/>  
MPRA Paper No. 71225, posted 13 May 2016 04:38 UTC

# The perils of first-order conditions of New Keynesian models

Minseong Kim

2016/05/11

## Abstract

This paper presents a case where first-order conditions used for the Calvo sticky-price New Keynesian model is insufficient and in fact results in calculation of a wrong equilibrium.

## 1 Calvo model

The Calvo sticky-price model [1] presented here has the same household optimization problem as before except that the budget constraint is now

$$P_t C_t + \frac{B_t}{R_t} \leq W_t N_t + D_t + B_{t-1} \quad (1)$$

Thus real wage first-order condition is given by

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi \quad (2)$$

Again assume  $P_0 = 1$ .

In the Calvo model, production function for an individual firm remains the same, but now price dispersion affects the final output. Production function will be changed to:

$$Y_t = s_t A_t N_t^{1-\alpha} \quad (3)$$

For the Calvo model,  $0 < s_t \leq 1$ , and no one value can be pre-determined for  $s_t$  without specified monetary policy.

Decreasing returns to scale is required for what follows - in constant returns to scale, the method presented below would fail, as will be seen.

I will assume  $P_0 = 1$ .

The household has utility function of

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \quad (4)$$

and tries to maximize intertemporal utility of

$$V = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (5)$$

But intertemporal nature will not matter for this paper, and thus focus will be given to  $U$ , instead of  $V$ .

Here,  $C_u$  is the level that brings the maximum utility to the household given  $C_u = s_t A_t N_u^{1-\alpha}$ .

Now for simplification, suppose  $\sigma \rightarrow 0$ ,  $\varphi \rightarrow 0$ . Technically, these values are greater than 0, but the equilibrium will not deviate too much from when one assumes  $\sigma = 0$ ,  $\varphi = 0$ .

In such a case, the limiting utility function is given by

$$U(C_0, N_0) = C_0 - N_0 \quad (6)$$

Let  $A_0 = 1$ . Then,  $N_0 = \left(\frac{C_0}{s_0}\right)^{\frac{1}{1-\alpha}}$ . Let  $s_0 \rightarrow 1$  for another simplification. Then  $N_0 = C_0^{\frac{1}{1-\alpha}}$ . Let us find out  $C_u$  by solving first-order condition of maximizing  $U$  (here, first-order condition will be OK).

$$C_u = (1 - \alpha)^{\frac{1-\alpha}{\alpha}} \quad (7)$$

$$C_u = x W_u N_u = x C_u^{\sigma + \frac{1+\varphi}{1-\alpha}} = x C_u^{\frac{1}{1-\alpha}} \quad (8)$$

$$x = \frac{1}{1 - \alpha} \quad (9)$$

$$N_u = (1 - \alpha)^{\frac{1}{\alpha}} \quad (10)$$

Let  $C_0 = k C_u$ , and let  $k' = k^{1/(1-\alpha)}$ . Then,

$$D_0 = C_0 - W_0 N_0 = C_0 - C_0^{\sigma} N_0^{1+\varphi} = C_0 - N_0 = k C_u - k' C_u^{\frac{1}{1-\alpha}} \quad (11)$$

$$W_0 N_u = N_u = C_u^{\frac{1}{1-\alpha}} \quad (12)$$

$$W_0 N_u + D_0 = k C_u - k' C_u^{\frac{1}{1-\alpha}} + C_u^{\frac{1}{1-\alpha}} \quad (13)$$

Suppose  $k \approx 0$  (I will simply set it as  $k = 0$ ), and let  $0 < q < 1$ . It is wished that  $q C_u \leq W_0 N_u + D_0$ . Does this hold?

$$q C_u \leq k C_u - k' C_u^{\frac{1}{1-\alpha}} + C_u^{\frac{1}{1-\alpha}} \quad (14)$$

$$q \leq k - k' C_u^{\frac{\alpha}{1-\alpha}} + C_u^{\frac{\alpha}{1-\alpha}} \quad (15)$$

$$q - k \leq C_u^{\frac{\alpha}{1-\alpha}} - k' C_u^{\frac{\alpha}{1-\alpha}} \quad (16)$$

Substituting the  $C_u$  equation,

$$q - k \leq (1 - \alpha)(1 - k') \quad (17)$$

with  $k = 0$

$$q \leq 1 - \alpha \quad (18)$$

Recall utility function:

$$T(C) = C - C^{\frac{1}{1-\alpha}} \quad (19)$$

Derivative of  $T$  at  $0 < C < 1$  satisfies  $0 < T'(C)$ . Thus, Equation 18 proves that there exists a case where the equilibrium thought to be allowed by New Keynesian modelling is not actually an equilibrium.

## 2 Conclusion

This paper suggests that first-order conditions derived from optimization problems are not sufficient to find Calvo New Keynesian model equilibrium.

## References

- [1] Calvo, G. A. (1983). “Staggered prices in a utility-maximizing framework”, *Journal of Monetary Economics* 12 (3): 383–398.